A NUMERICAL STUDY OF THE FLOW OF GRANULAR MATERIALS BETWEEN TWO VERTICAL FLAT PLATES WHICH ARE AT DIFFERENT TEMPERATURES

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ABSTRACT

The present study considers an assembly of spherical particles, densely packed, between two vertical flat plates which are at different temperatures. A continuum model is used and the flow due to such a temperature difference is investigated. For a fully developed flow of these materials, the governing equations reduce to a system of coupled, non-linear ordinary differential equations. The equations are integrated numerically. In so doing, the equations are discretized using the conventional central finite difference approximation technique and the unknown conditions are assumed to reduce the problem to an initial value problem. After the trial solutions the known boundary conditions at the edge of the integration interval are compared with the correspondent values provided by the trial solutions. If the solutions at this point do not agree with the known boundary conditions the Newton-Raphson method is used to correct the initial guesses and the iteration is repeated. This procedure continues until the solutions converge to the given values. The technique is found to be powerful for this type of application.

1. INTRODUCTION

Granular materials are used in many engineering processes such as the handling of coals, agricultural products, dry chemicals, sand, and other particulate solids. For a thorough and up-to-date review of flowing granular materials the reader is referred to a recent article by Hutter and Rajagopal (1994). While heat transfer processes play an important role in the handling and the processing of these materials (Gudhe et al., (1994), thermal convection problems have received little attention. Of course, for the cases that have been studied, a

thermodynamic analysis of the constitutive equations has not been carried out.

In many applications these materials are also heated prior to processing, or cooled after processing (Patton et al., 1986, Uhl and Root, 1967, Gudhe et al., 1994, Sullivan and Sabersky, 1975, Spelt et al. 1982). Granular materials exhibit phenomena like normal stress differences in a simple shear flow, a phenomena usually referred to as *dilatancy* (Reynolds, 1886). The normal-stress phenomenon is a characteristics of non-Newtonian fluids and non-linear elastic solids. One approach in the modeling of granular materials is to treat it as a *continuum*, which assumes that the materials properties of the ensemble may be represented by continuous functions. One of the early continuum models for flowing granular materials based on the principles of modern continuum mechanics was proposed by Goodman and Cowin (1971,1972). This work was subsequently modified and improved upon by other investigators (Savage, 1984, Ahmadi, 1982). Another approach used in the modeling of granular materials is based on the techniques used in the *kinetic theory* of gases (Boyle and Massoudi, 1990).

In this work we consider the natural convection of granular materials. The material is between two infinite vertical flat plates which are at two different temperatures, (cf. Figure 1). Using a continuum formulation for the stress tensor, the governing equations for the conservation of mass, momentum, and energy are developed. A very basic formulation of this problem was given in Massoudi et al. (1994).
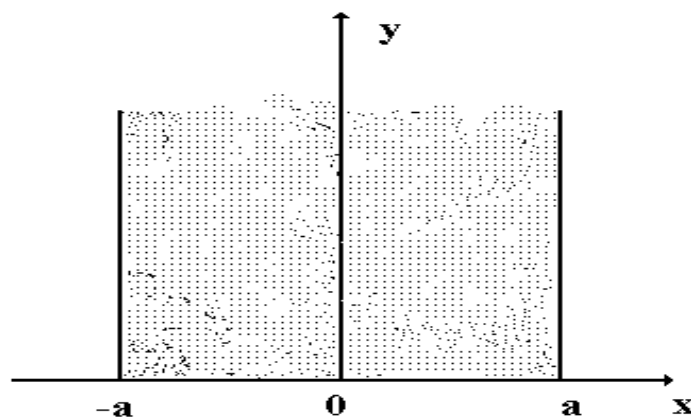

Figure 1.   A sketch of the theoretical model

2. GOVERNING EQUATIONS:

We assume that the stress tensor **T** is given by, (Goodman and Cowin, 1971, Rajagobal and Massoudi, 1990).

$$\mathbf{T} = \{\beta_o(v) + \beta_1(v)\, \nabla v . \nabla v + \beta_2(v)\, tr\, \mathbf{D}\}\mathbf{1} + \beta_4(v)\nabla v \otimes \nabla v + \beta_3(v)\mathbf{D} \tag{1}$$

where v denotes the volume fraction of the particles, **D** is the symmetric part of the velocity gradient, $\beta_o(v)$ is similar to pressure in a compressible fluid and is given by an equation of state, $\beta_2(v)$ is akin to the second coefficient of viscosity in a compressible fluid, $\beta_1(v)$ and $\beta_4(v)$ are the material parameters that reflect the distribution of the granular materials and $\beta_3(v)$ is the viscosity of the granular materials. In general, the material properties β_o through β_4 are functions of the density (or volume fraction v), temperature, and the principal invariant of the tensor **D**, given by:

$$\mathbf{D} = \frac{1}{2}[(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T] \tag{2}$$

where **u** is the velocity vector of the particles. Furthermore, v is related to the bulk density of the material ρ, through $\rho = \rho_s v$. Where ρ_s is the actual density of the grains at the place **x** and time t and the field v is called the volume fraction (or the volume distribution) and is related to porosity n or the void ratio e by

$$v = 1 - n = \frac{1}{1 + e} \tag{3}$$

The material properties β_o through β_4 are discussed by Gudhe et al., (1994). Furthermore, we assume that the heat flux vector **q** satisfies Fourier's law, i.e.,

$$\mathbf{q} = -K\nabla\theta \tag{4}$$

where θ is the temperature and K is the thermal conductivity, which, in general, is a function of v and θ. The governing equations of mass, momentum, and energy are.

$$\frac{\partial\rho}{\partial t} + div(\rho\mathbf{u}) = 0 \tag{5}$$

$$\rho\frac{d\mathbf{u}}{dt} = div\,\mathbf{T} + \rho\mathbf{b} \tag{6}$$

$$\rho\frac{d\varepsilon}{dt} = \mathbf{T.L} - div\,\mathbf{q} + \rho r \tag{7}$$

where **b** is the body force vector, ε denotes the specific internal energy, **q** is the heat flux vector, r is the radiant heat, and **L** is the velocity gradient. For the problem under consideration, we make the following assumptions: (i) the motion is steady; (ii) radiant heating r can be ignored; (iii) the constitutive equation for the stress tensor is given by equation (1) and the constitutive equation for the heat flux vector is that of Fourier's law,

given by equation (4); (iv) the density, velocity and temperature fields are of the form

$$v = v(x); \quad \mathbf{u} = u(x)\mathbf{j}; \quad \theta = \theta(x); \quad \rho \mathbf{b} = -\rho_s v[1 - \gamma(\theta - \theta_m)]g\mathbf{j} \tag{8}$$

where θ_m is a reference temperature, (for example $\theta_m = (\theta_1 + \theta_2)/2$), g is the acceleration due to gravity, and γ is the coefficient of thermal expansion, and $\mathbf{j}$ is the unit vector in the y-direction. Furthermore, we assume

$$\beta_o(v) = kv; \quad \beta_3(v) = \beta_3^*(v + v^2) \tag{9}$$

where β_3^* is a constant and β_1, β_2, β_4 are assumed to be constant. For thermal conductivity, we assume (Bashir and Goddard, 1990, Batchelor and O'Brien, 1977).

$$K = K_m(1 + 3\zeta v) \tag{10}$$

where $\zeta = (\psi - 1)/(\psi + 1)$. Here, ψ is the ratio of conductivity of the particle to that of the matrix, and K_m is the thermal conductivity of the matrix. With the above assumptions, we obtain the following equations

$$k \frac{dv}{dx} + 2(\beta_1 + \beta_2) \frac{dv}{dx} \frac{d^2v}{dx^2} = 0 \tag{11}$$

$$\beta_3^*(v + v^2) \frac{d^2u}{dx^2} + \beta_3^*(1 + 2v) \frac{dv}{dx} \frac{du}{dx} = \rho_s v[1 - \gamma(\theta - \theta_m)]g \tag{12}$$

$$2K_m(1 + 3\zeta v) \frac{d^2\theta}{dx^2} + 6K_m \zeta \frac{dv}{dx} \frac{d\theta}{dx} = -\beta_3^*(v + v^2)(\frac{du}{dx})^2 \tag{13}$$

with $u = 0$, $\theta = \theta_1$ at $x = -a$; and $u = 0$, $\theta = \theta_2$ at $x = a$, and $\theta_1 > \theta_2$. For the volume fraction, we use the symmetry condition and mass-averaged quantity (see Gudhe et al., 1994) and assume the following

$$(\frac{dv}{dx})_{X=0} = 0; \quad Q = \int_{-a}^{a} vdx \tag{14}$$

Where Q is given. It is obvious from Eq. (11) that $dv/dx = 0$ or $v = $ constant is a solution to that equation.

3. NUMERICAL SOLUTIONS

Let's define ξ, Φ, and Γ by the following equation:

$$\xi = \frac{x}{a} \; ; \quad \phi = \frac{u}{U_o} \; ; \quad \Gamma = \frac{\theta - \theta_m}{\theta_1 - \theta_2} \tag{15}$$

where U_o is a reference velocity. Then equations (11) to (13) become

$$\frac{d^2v}{d\xi^2} + R_1 = 0 \tag{16}$$

$$R_2(v + v^2)\frac{d^2\phi}{d\xi^2} + R_2(1 + 2v)\frac{dv}{d\xi}\frac{d\phi}{d\xi} - v + R_4 v\Gamma = 0 \tag{17}$$

$$(1 + 3\zeta v)\frac{d^2\Gamma}{d\xi^2} + 3\zeta\frac{dv}{d\xi}\frac{d\Gamma}{d\xi} + R_3 (v + v^2) [\frac{d\phi}{d\xi}]^2 \tag{18}$$

with $\phi = 0$ and $\Gamma = 1/2$ at $\xi = -1$, and $\phi = 0$; $\Gamma = -1/2$ at $\xi = 1$, and

$$(\frac{dv}{d\xi})_{\xi=0} = 0; \quad N = \int_{-1}^{1} v d\xi \tag{19}$$

where $N = Q/a$. The dimensionless parameters R_1, R_2, R_3, and R_4 are given by

$$R_1 = \frac{ka^2}{2(\beta_1 + \beta_4)} \; ; \quad R_2 = \frac{\beta_3^* U_o}{2a^2\rho_s g} \tag{20}$$

$$R_3 = \frac{\beta_3^* U_o^2}{2K_m(\theta_1 - \theta_2)}; \quad R_4 = \gamma(\theta_1 - \theta_2) \tag{21}$$

These dimensionless parameters have the following physical meaning: R_1 is the ratio of the pressure force to the force developed in the material due to the distribution of the void. R_2 is the ratio of the viscous force to the gravity force and R_3 is the product of the Prandtl number and the Eckert number.

With the conditions given by Eq. (19), the solution for the volume fraction is obtained as

$$v = -\frac{R_1\xi^2}{2} + (\frac{Q}{a} + \frac{R_1}{3})\frac{1}{2} \tag{22}$$

Equations . (17), and (18)are discretized using the following equation

$$\frac{d^2()}{d\xi^2} = \frac{()^{(i+1)} - 2()^i + ()^{(i-1)}}{\Delta\xi^2}; \quad \frac{d()}{d\xi} = \frac{()^{(i+1)} - ()^{(i-1)}}{2\Delta\xi} \tag{23}$$

Where () represents Γ, and ϕ. Superscripts (i+1), (i), and (i-1) denote the values of ϕ and Γ at $(\xi+\Delta\xi)$, (ξ), and $(\xi-\Delta\xi)$, respectively. Substituting these expressions into Eqs. (17) and (18) we obtain the solutions for ϕ and Γ at $\xi+\Delta\xi$ as

$$\phi^{(i+1)} = \frac{v^{(i)} - R_4 v^{(i)} \Gamma^{(i)} + 2\Pi_1\phi^{(i)} + (\Pi_2 - \Pi_1) \phi^{(i-1)}}{\Pi_2 + \Pi_1} \tag{24}$$

$$\Gamma^{(i+1)} = \frac{-\Pi_5[\phi^{(i+1)} - \phi^{(i-1)}]^2 + [\Pi_4 - \Pi_3]\Gamma^{(i-1)} + 2\Pi_3\Gamma^{(i)}}{\Pi_4 + \Pi_3} \tag{25}$$

$$\Pi_1 = \frac{R_2(v + v^2)}{\Delta\xi^2}; \quad \Pi_2 = \frac{R_2 (1 + 2v)}{2\Delta\xi} \frac{dv}{d\xi} \tag{26}$$

$$\Pi_3 = \frac{1 + 3\zeta v}{\Delta\xi^2}; \quad \Pi_4 = 3\zeta \frac{dv}{d\xi}; \quad \Pi_5 = \frac{R_3(v + v^2)}{4\Delta\xi^2} \tag{27}$$

where i is from 0 (for $\xi = -1$) to N (for $\xi = 1$) with

$$\phi^{(i=0)} = \phi^{(i=N)} = 0; \quad \Gamma^{(i=0)} = 1/2; \quad \Gamma^{(i=N)} = -1/2 \tag{28}$$

and from Eq. (23), $\phi^{(-1)}$ and $\Gamma^{(-1)}$ are given by

$$()^{(-1)} = ()^{(2)} - 2\Delta\xi(\frac{d()}{d\xi})_{at\ \xi = -1} \tag{29}$$

where ()is for ϕ, and Γ. Since $d\phi/d\xi$ and $d\Gamma/d\xi$ at $\xi = -1$ are not known, they must be assumed so that $\phi^{(2)}$, $\Gamma^{(2)}$ to $\phi^{(N-1)}$, $\Gamma^{(N-1)}$ can be calculated. After the trial solutions are obtained, the known boundary conditions of ϕ and Γ at $\xi = 1$ are compared with the corresponding values provided by the trial solutions. If the solutions at this point do not agree with the known boundary conditions another guess must be used and the iteration is

repeated. This procedure continues until the solutions for ϕ and Γ provided by the initial guess conditions converge to the given values. Let X and Y be the values for the unknown conditions such that

$$(\frac{d\phi}{d\xi})_{\xi=-1} = X; \quad (\frac{d\Gamma}{d\xi})_{\xi=-1} = Y \tag{30}$$

The problem is to find the values of X and Y to be such that the boundary conditions at $\xi = 1$ are satisfied, that is to find the solutions of

$$\phi_{(at\ \xi=1)}[X,Y] = 0; \quad \Gamma_{(at\ \xi=1)}[X,Y] = -1/2 \tag{31}$$

Thus, if the guess values X and Y are such that these equations are not satisfied, the necessary corrections ΔX for X and ΔY for Y must be calculated. By using the Newton-Raphson technique, the necessary corrections ΔX and ΔY to the first approximation are obtained from the solution of the following set of equations (Phuoc and Durbetaki, 1985):

$$\phi + \frac{\partial\phi}{\partial X}\Delta X + \frac{\partial\phi}{\partial Y}\Delta Y = 0; \quad \Gamma + \frac{\partial\Gamma}{\partial X}\Delta X + \frac{\partial\Gamma}{\partial Y}\Delta Y = -\frac{1}{2} \tag{32}$$

Let the subscript X denote the partial derivative with respect to X, $(\partial/\partial X)$, and subscript Y denote the partial derivative with respect to Y, $(\partial/\partial Y)$ and let

$$\delta_1 = \phi + \phi_X\Delta X + \phi_Y\Delta Y - \phi_{(at\ \xi=1)} \tag{33}$$

$$\delta_2 = \Gamma + \Gamma_X\Delta X + \Gamma_Y\Delta Y - \Gamma_{(at\ \xi=1)} \tag{34}$$

Values of ΔX and ΔY should be such that the sum of $\delta = (\delta_1^2 + \delta_2^2)$ is a minimum. Thus, let $d\delta/d(\Delta X) = d\delta/d(\Delta Y) = 0$, the corrections ΔX and ΔY are calculated using the following equations:

$$a_{11}\Delta X + a_{12}\Delta Y = -a_{13} \quad \text{and} \quad a_{21}\Delta X + a_{22}\Delta Y = -a_{23} \tag{35}$$

where a_{11} through a_{23} are given as

$$a_{11} = (\phi_X)^2 + (\Gamma_X)^2; \quad a_{12} = \phi_X\phi_Y + \Gamma_X\Gamma_Y \tag{36}$$

$$a_{13} = \Gamma_X/2 + \phi\phi_X + \Gamma\Gamma_X; \quad a_{21} = a_{12} \tag{37}$$

$$a_{22} = (\Gamma_Y)^2 + (\phi_Y)^2 \; ; \quad a_{23} = \Gamma_Y/2 + \phi\phi_Y + \Gamma\Gamma_Y \tag{38}$$

Values of $a_{11}...a_{23}$ are obtained after the partial derivatives ϕ_X; ϕ_Y, Γ_X, and Γ_Y are calculated at $\xi = 1$. These partial derivatives come from the differentiation of the Eqs. (24) and (25) with respect to X and Y as follows:

$$\phi_J^{(i+1)} = \frac{-R_4 \, v^{(i)} \, \Gamma_J^{(i)} + 2\Pi_1\phi_J^{(i)} + (\Pi_2 - \Pi_1) \, \phi_J^{(i-1)}}{\Pi_1 + \Pi_2} \tag{39}$$

$$\Gamma_J^{(i+1)} = \frac{f(\phi_J) + (\Pi_4 - \Pi_3) \, \Gamma_J^{(i-1)} + 2\Pi_3 \, \Gamma_J^{(i)}}{\Pi_3 + \Pi_4} \tag{40}$$

$$f(\phi_J) = 2\Pi_5 \, (\phi^{(i+1)} - \phi^{(i-1)})(\phi_J^{(i+1)} - \phi_J^{(i-1)}) \tag{41}$$

Where J is for X and Y. Equations (39) to (41) are called perturbation equations which give the rate of change of the solutions of the original equations, (Eq.24 and Eq.25), with respect to the initially guessed conditions. These equations are calculated with the following initial conditions of $d\phi/d\xi$ and $d\Gamma/d\xi$ at $\xi = -1$ as follows:

$$(\frac{d\phi}{d\xi})_X = (\frac{d\Gamma}{d\xi})_Y = 1; \quad (\frac{d\phi}{d\xi})_X = (\frac{d\Gamma}{d\xi})_Y = 1 \tag{42}$$

$$\phi_X^{(-1)} = \phi_X^{(2)} - 2\Delta\xi; \quad \Gamma_Y^{(-1)} = \Gamma_Y^{(2)} - 2\Delta\xi \tag{43}$$

$$\phi_Y^{(-1)} = \phi_X^{(1)} = \phi_Y^{(1)} = \Gamma_X^{(-1)} = \Gamma_X^{(1)} = \Gamma_Y^{(1)} \tag{44}$$

Equations (24), (25), (39-41) are calculated simultaneously using $\Delta\xi = 0.1$. The convergence was obtained after 5 to 6 iterations.

4. RESULTS AND DISCUSSIONS

When granular materials are packed between two vertical plates, which are at different temperatures, the materials near the hot plate are heated and the materials near the colder plate are cooled. As a result of such a temperature difference a natural convection flow ensues and the materials move. It is expected that such a flow depends on the competition between various forces such as the pressure force, the viscous force, the gravity force, the
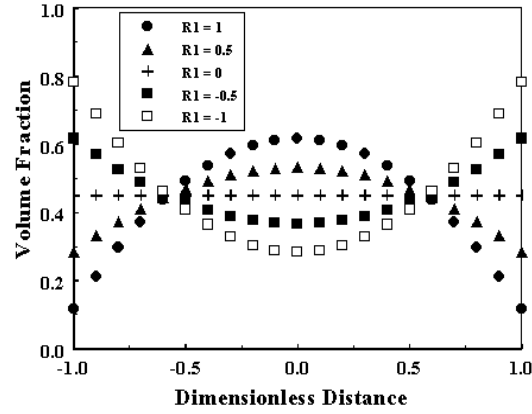
Figure 2.   Volume fraction ν as a function of ξ: Effect of R_1; (N = 0.9)

force developed in the materials etc. and the transport properties. Thus, the solutions of the above equations with respect to R_1, R_2, R_3, R_4, N, and ς as parameters will reveal the characteristics of the natural convection flow and heat transfer between the hot and the cold surfaces. Since the main emphasis in this paper is on the numerical technique developed for the calculations, we present a few cases showing the powerful capability of the technique in obtaining the solutions for this complex problem. In particular, we focus on the importance of the effects of R_1 and R_3 as typical examples for such an application.

For a given value of N, equation (22) describes the distribution of the volume fraction in ξ-direction under the competition between the pressure force and the force developed in the materials due to the distribution of the particles (or the volume fraction forces). Such a competition is represented by the dimensionless parameter R_1 which was defined as the ratio of the pressure force to the force developed in the materials due to the volume fraction distribution. Typical results showing the effect of the dimensionless parameter R_1 on the distribution of the volume fraction were plotted in Fig. 2 for N = 0.9 and R_1 was taken from -1.0 to 1.0. These values of R_1 were chosen so that two possible patterns of the distribution of the materials could be demonstrated.

The results showed that the volume fraction ν as a function of ξ decreased as the absolute value of R_1 decreased and it became constant and equal to N/2 when R_1 was equal to zero. It should be noted that R_1 is large when the pressure force is dominant over the force developed in the materials and R_1 is small when the force developed in the materials is large compared to the pressure force. Thus, the dependency of ν on R_1 presented here indicates that these forces have two different functions. The pressure force is the force that tends to disturb the distribution of the materials while the other can be thought of as the force that is acting against the pressure force and tends to rearrange the materials into a more uniform distribution pattern.
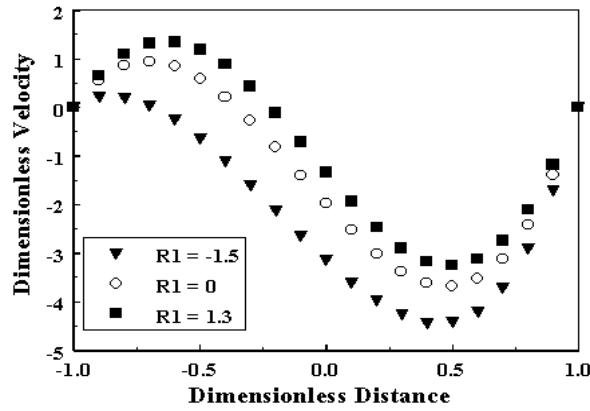
Figure 3.   Dimensionless velocity u as a function of the
              dimensionless distance ξ: Effect of R_1; (N = 0.9; R_2
              = 0.1; R_3 = 0.01; R_4 = 10; ζ = 0.25)

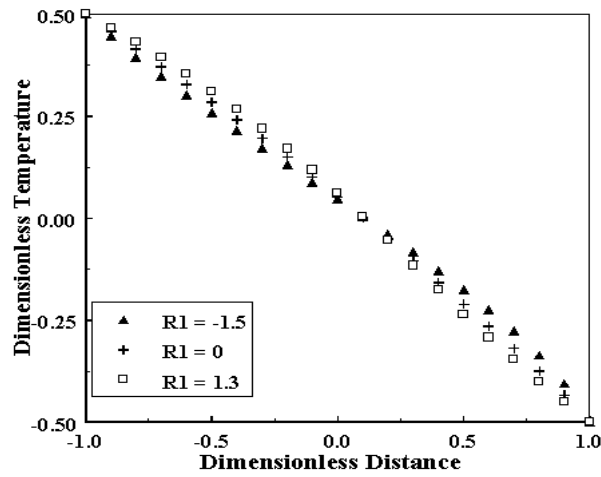

Figure 4.   Dimensionless temperature Γ as a function of the
              dimensionless distance ξ: Effect of R_1; (N = 0.9; R_2
              = 0.1; R_3 = 0.01; R_4 = 10; ζ = 0.25)

When R_1 was not equal to zero, however, two distribution patterns were obtained depending
on whether R_1 was negative or positive. When R_1 was positive (the pressure force has the
same sign as the force developed in the materials) v reached its maximum value at $\xi = 0$ and
its minimum values were at the plates. Thus, this means that the region near the center has
more particles than do the regions near the plates. When R_1 was negative (the pressure force
has the opposite sign to the force developed in the materials) v had a minimum value at $\xi =$

0 and maximum values were at the plates. More particles are packed near the plates than the region near the center.

The effects of R_1 on the velocity and temperature profiles were shown in Figs. 3 and 4 respectively. The results shown in these figures were obtained when R_2, R_3, R_4, and N were kept constant and R_1 =-1.5, 0.0 and 1.3 were used. These values were chosen to represent three different patterns of the distribution of the volume fraction. Negative R_1, (R_1 =-1.5) represents the situation under which the concentration of the materials is denser near the plates than in the region near the centerline. Positive R_1, (R_1= 1.3), is for the case that there are more materials in the center region than in the regions near the plates. And R_1 = 0 represents the case that the materials are uniformly distributed.

For these three situations the results always showed a nearly linear decrease of the dimensionless temperature from the hot plate to the colder one. Thus, the dimensionless parameter R_1 does not have a significant effect on the temperature distribution. The results on the velocity profiles exhibited the fact that the materials moved but the patterns by which the materials moved were the same regardless of how the volume fraction behaved. That is the materials always moved with forward velocities in the region near the hotter plate and reserved velocities in the region near the colder plate. The only effect of R_1 that was detected by this work was that, when R_1 had a negative value and its absolute value increased the forward flow velocity decreased and the reversed flow velocity increased. On the other hand, when R_1 had a positive value the forward flow velocity increased and the reversed flow velocity decreased with R_1.
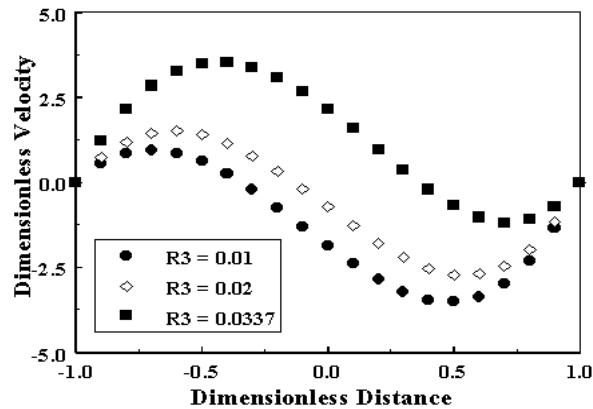

Figure 5.   Effect of the dimensionless parameter R_3 on the dimensionless velocity profiles; (N = 1; R_1 = 0; R_2 = 0.1; R_4 = 10; ζ = 0.25)
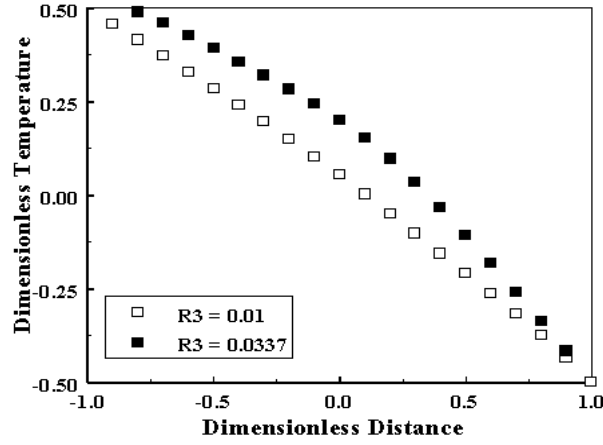
Figure 6.　　Effect of the dimensionless parameter R_3 on the dimensionless temperature profiles;　$(N = 1; R_1 = 0; R_2 = 0.1; R_4 = 10; \zeta = 0.25)$

The effects of the dimensionless parameter R_3 on the velocity and the temperature profiles were shown in Figs. 5 and 6 respectively. The results indicated that the forward flow was dominant when R_3 was large while the reversed flow was significant when R_3 was small. Parameter R_3 also had a strong effect on the material heating process. When R_3 was small the heat transfer at the hot plate $(\xi=-1)$ represented by $(d\Gamma/d\xi)_{\xi=-1}$ had negative values. Thus the hot plate acted as a heat source that heated the materials. When R_3 was large, however, the heat transfer at $\xi=-1$ reversed its sign and $(d\Gamma/d\xi)_{\xi=-1}$ became positive. The hot plate became a heat sink and the heat was transferred from the materials to the plate.

The materials between the two plates are heated by two sources: the high temperature of the hot plate and the heat generated by the flow of the materials. If the material flow is not significant, the hot plate becomes a significant heat source that heats the materials. In this case the material temperature must be less than that of the hot plate and $d\Gamma/d\xi$ is negative at $\xi=-1$. If the material flow is intense, the heat generated (or the viscous dissipation) from such a flow can be more significant than the heat transfer from the hot plate. In this case, the materials can be heated to a temperature that is higher than the temperature of the plate. The materials is now a heat source with respect to the hot plate. Heat is transferred from the materials to the plate and $d\Gamma/d\xi$ will have a positive value at $\xi=-1$.

5. CONCLUSIONS

The natural flow and heat transfer of an assembly of spherical particles, densely packed,

between two vertical flat plates which are at different temperatures have been studied. A continuum model was used to develop the governing equations. For a fully developed flow of these materials, the governing equations reduced to a system of coupled, non-linear ordinary differential equations. Various dimensionless parameters were defined to represent the effects of various transport properties and the competitions between the pressure force, the volume fraction distribution forces, and the gravity forces. The results showed that the materials moved with forward velocities near the hot plate and reversed velocities near the colder plate

The present numerical calculations used the conventional central finite difference approximation technique and the Newton-Raphson method for correcting the initial guesses for the unknown conditions. The technique required an additional set of N_{ode} x $N_{unknown}$ (N_{ode} is the number of the governing equations and $N_{unknown}$ is the number of the unknown boundary conditions) differential equations that must be integrated simultaneously with the original governing equations. These equations, referred as the perturbation equations, gave the rate of change of the solutions of the original differential equations with respect to the guessed conditions. Thus, the technique reduced the guess work and the computer time and it was powerful for this type of application. The convergence was obtained after less than five or six iterations and it was insensitive to the initial guess for small values of R_3. When larger values of R_3 were used the convergence was more difficult and it became very sensitive to the initial guess.

6. NOMENCLATURES

b　denotes the body force vector
D　is the symmetric part of the velocity gradient, Eq. 4
e　void ratio e
g　is the acceleration due to gravity
K　thermal conductivity
K_m　is the thermal conductivity of the matrix.
L　is the velocity gradient
n　porosity n
Q　defined by Eq. 14
q　is the heat flux vector
r　radiant heat
R_1　defined by Eq. 20
R_2　defined by Eq. 20
R_3　defined by Eq. 21
R_4　defined by Eq. 21
T　is the stress tensor
t　time
U_o　reference velocity
u　is the x-component velocity of the particles.
u　is the velocity vector of the particles

x	x-coordinate
β_o	is similar to pressure in a compressible fluid
β_2	is akin to the second coefficient of viscosity in a compressible fluid
β_1, β_4	are the material parameters that reflect the distribution of the granular materials
β_3	is the viscosity of the granular materials
β_3^*	is a constant
Γ	dimensionless temperature
γ	is the coefficient of thermal expansion.
ε	denotes the specific internal energy
ζ	is defined as $(\psi-1)/(\psi+1)$
θ_m	is the reference temperature, $\theta_m=(\theta_1+\theta_2)/2$
θ	is the temperature
ν	volume fraction of the particles
ξ	dimensionless distance
ρ	is the actual density of the grains
ϕ	dimensionless velocity
ψ	is the ratio of conductivity of the particle to that of the matrix

Subscripts
X	derivative with respect to X
Y	derivative with respect to Y

Superscripts
(i)	at i position
(i-1)	at i-1 position
(i+1)	at i+1 position

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